

# Can Symmetric Texture reproduce Neutrino Bi-large Mixing?

Masako BANDO <sup>1</sup> \*, Satoru KANEKO <sup>2</sup> †, Midori OBARA <sup>3</sup> ‡

and

Morimitu TANIMOTO <sup>2</sup> §

<sup>1</sup> *Aichi University, Aichi 470-0296, Japan*

<sup>2</sup> *Department of Physics, Niigata University, Niigata, 950-2128, Japan*

<sup>3</sup> *Institute of Humanities and Sciences,  
Ochanomizu University, Tokyo 112-8610, Japan*

## Abstract

We study the symmetric texture of geometric form with 2-zeros to see if it is consistent with the presently-known neutrino masses and mixings. In the neutrino mass matrix elements we obtain numerically the allowed region of the parameters including CP violating phases, which can reproduce the present neutrino experiment data. The result of this analysis dictates the narrow region for the GUT model including Pati-Salam symmetry with texture zeros to be consistent with the experimental data. The  $|U_{e3}|$  and  $J_{CP}$  are also predicted in such models.

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\*E-mail address: bando@aichi-u.ac.jp

†E-mail address: kaneko@muse.sc.niigata-u.ac.jp

‡E-mail address: midori@hep.phys.ocha.ac.jp

§E-mail address: tanimoto@muse.sc.niigata-u.ac.jp

Neutrino experiments by Super-Kamiokande [1, 2] and SNO[3] have brought us an outstanding fact on the neutrino oscillation. Recent results from KamLAND have almost confirmed the large neutrino mixing solution that is responsible for the solar neutrino problem nearly uniquely [4]. We have now common information concerning the neutrino mass difference squared ( $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\text{sun}}^2$ ) and neutrino flavor mixings ( $\sin^2 2\theta_{\text{atm}}$  and  $\tan^2 \theta_{\text{sun}}$ ) [5] as follows:

$$\begin{aligned} 0.35 \leq \tan^2 \theta_{12} \leq 0.54, \quad & 6.1 \times 10^{-5} \leq \Delta m_{\text{sun}}^2 \leq 8.3 \times 10^{-5} \text{ eV}^2, \quad 90\% \text{C.L.} \\ 0.90 \leq \sin^2 2\theta_{23}, \quad & 1.3 \times 10^{-3} \leq \Delta m_{\text{atm}}^2 \leq 3.0 \times 10^{-3} \text{ eV}^2, \quad 90\% \text{C.L.} \end{aligned} \quad (1)$$

In these data it is remarked that the neutrino mixing is the bi-large and the ratio  $\Delta m_{\text{sun}}^2 / \Delta m_{\text{atm}}^2$  is  $\sim \lambda^2$  with  $\lambda \simeq 0.2$ . A constraint has also been placed on the third mixing angle from the reactor experiment of CHOOZ [6]. These results are very important for model buildings of flavors.

There are many attracting points in grand unified theories (GUT), anomaly cancellation between quarks and leptons in one family, gauge coupling unification, electromagnetic charge quantization, e.t.c.. In the framework of GUT, quarks and leptons are unified in some way and their masses and mixing angles are mutually related. Now the neutrino sector which shows less hierarchical and bi-large mixing angles is quite different from the quark sector where far stronger hierarchy is observed with very tiny mixing angles. So the problem is whether such large difference of quark and lepton sectors can be consistent with GUT. So far as we assume general  $U(1)$  family structure [7] with order 1 coefficients of Yukawa couplings, the simplest example of symmetric mass matrix is already excluded because the resultant neutrino mass matrix is predicted to be also hierarchical with small mixings. However if we assume some additional symmetry to protect some components of the mass matrix leading "zero" texture, the above statement is no more guaranteed [8]. Actually in the previous paper [9] an example of symmetric 4 zero texture is shown to reproduce the bi-large neutrino mixing compatible with GUT. On the other hand, the experimental data already dictates the desired form of neutrino mass matrix  $M_\nu$  for which the order of each component is as follows [10];

$$M_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} m_\nu \quad (2)$$

Note that, in order for the above form to reproduce the bi-large mixing with the observed mass squared differences, it is not sufficient to discuss only the order of magnitudes, and we have to tune the coefficients very carefully. The minimum texture preserving the above properties would be the one having some zeros [11, 12, 13, 14, 15], where we need the 23 element of order 1 to get large 23 mixing angle, and further the determinant of the  $2 \times 2$  matrix of the right bottom corner should become of order  $\lambda$  in order to reproduce the experimental mass difference ratio  $\Delta m_{\text{sun}}^2/\Delta m_{\text{atm}}^2$ , the 22 element should be of order 1. Also the 12 (13) element must be non-zero to reproduce large mixing angle  $\theta_{12}$ . So the only possible zeros are for 11 and 13 (12) elements namely two-zero symmetric texture. Thus we can take the simplest form of neutrino mass matrix at GUT scale as a minimal model \* including a phase  $\phi$ ;

$$M_\nu = m_\nu \begin{pmatrix} 0 & \beta & 0 \\ \bar{\beta} & \bar{\alpha} & \bar{h} \\ 0 & \bar{h} & 1 \end{pmatrix} = m_\nu P_\nu^T \begin{pmatrix} 0 & \beta & 0 \\ \beta & e^{i\phi}\alpha & h \\ 0 & h & 1 \end{pmatrix} P_\nu, \quad \begin{aligned} \beta &\simeq O(\lambda) \\ \alpha &\simeq O(1) \\ h &\simeq O(1) \end{aligned} \quad (3)$$

with  $\bar{\alpha}, \bar{\beta}, \bar{h}$ , being made positive real numbers,  $\alpha, \beta, h$  by factored out the phases by the diagonal phase matrix  $P_\nu^\dagger$ .

In this letter we investigate this kind of 2-zero texture including CP phase and examine parameter regions which are consistent with the present experiments. The neutrino and quark mixings are expressed by MNS [17] and CKM matrices, respectively,

$$U_{MNS} = U_l^\dagger U_\nu, \quad U_{CKM} = U_u^\dagger U_d, \quad (4)$$

which are further divided into two unitary matrices,  $U_u$  and  $U_d$  or  $U_l$  and  $U_\nu$ , respectively, which diagonalize the  $3 \times 3$  up and down quark mass matrices  $M_u$  and  $M_d$  or charged

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\*Another 2-zero texture has been adopted by Chen and Mahanthappa [16].

†This kind of 4-zero case has been studied extensively for the quark masses;

$$M_u = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & 1 \end{pmatrix} m_t, \quad M_d = \begin{pmatrix} 0 & A' & 0 \\ A' & B' & C' \\ 0 & C' & 1 \end{pmatrix} m_b.$$

Here the matrix is assumed to be factored out by  $P$  in the four-zero texture case, which is exactly possible in the case of 6-zero texture. Note that we cannot factor out all the phases to make the matrix elements of  $M$  all real and there remains one phase as is seen in Eq. (3).

lepton and neutrino mass matrices,  $M_l$  and  $M_\nu$  respectively;

$$U_l^\dagger M_l V_l = \text{diag}(m_e, m_\mu, m_\tau), \quad U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad (5)$$

$$U_u^\dagger M_u V_u = \text{diag}(m_u, m_c, m_t), \quad U_d^\dagger M_d V_d = \text{diag}(m_d, m_s, m_b), \quad (6)$$

where  $U$  and  $V$  are unitary matrix acting on left- and right-handed fermions, respectively and  $\text{diag}(m_i, m_j, m_k)$  are mass eigenvalues of relevant fermions. We assume that the neutrino masses are obtained from the so-called see-saw mechanism with huge right-handed Majorana masses ( $M_R$ ) and with the Dirac neutrino masses ( $M_{\nu_D}$ )

$$M_\nu = M_{\nu_D}^T M_R^{-1} M_{\nu_D}. \quad (7)$$

Generally large neutrino mixing angles may be derivable even in the case when the Dirac neutrino mass matrix shows strong hierarchical with very small mixing angles if  $M_R$  is tuned very properly <sup>‡</sup>. However here we try to find the conditions for reproducing the experiments without fine tuning.

Let us see how the parameters appearing in Eq. (3) at GUT scale are generally constrained from the present experimental neutrino data. For a moment forget about how to derive the parameters of  $M_\nu$  and just see how the parameter regions of  $h$  and  $\phi$  are constrained from the experimental data of  $\sin^2 2\theta_{\text{atm}}, \tan^2 \theta_{\text{sun}}$  and the ratio of  $\Delta m_{\text{sun}}^2$  to  $\Delta m_{\text{atm}}^2$  in terms of four parameters  $\alpha, \beta, h$  and  $\phi$ . To make numerical calculation more strictly, we must take account of the contributions from the charged lepton side,  $U_l$  in Eq. (4). The symmetric charged lepton mass matrix is written in terms of the real matrix  $(\overline{M}_l)_{RL}$  and further diagonalized to  $M_l^{\text{diag.}}$  by  $O_l$  [19];

$$\begin{aligned} (M_l)_{RL} &= P_l^T (\overline{M}_l)_{RL} P_l, & O_l^T \overline{M}_l O_l &= M_l^{\text{diag.}}, \\ &\rightarrow O_l^T (P_l^T)^{-1} M_l P_l^{-1} O_l &\equiv M_l^{\text{diag.}}. \end{aligned} \quad (8)$$

We use the following symmetric matrix having 2-zeros for  $\overline{M}_l$ ,

$$(\overline{M}_l)_{RL} \simeq \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}, \quad (9)$$

where  $m_e, m_\mu, m_\tau$  are charged lepton masses at  $M_{\text{GUT}}$  scale. Here, we ignore the RGE effect from  $M_{\text{GUT}}$  to  $M_R$  scale considering that it almost does not change the values of

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<sup>‡</sup>We call such cases "see-saw enhancement" [18].

masses for quarks and leptons. On the basis where the charged lepton mass matrix is diagonalized, the neutrino mass matrix at  $M_R$  scale is obtained from Eq. (3)

$$\tilde{M}_\nu(M_R) = O_l^T (P_l^{-1})^T P_\nu^T \overline{M}_\nu(M_R) P_\nu P_l^{-1} O_l , \quad (10)$$

where

$$\overline{M}_\nu(M_R) = \begin{pmatrix} 0 & \beta & 0 \\ \beta & e^{i\phi}\alpha & h \\ 0 & h & 1 \end{pmatrix} m_\nu , \quad Q \equiv P_\nu P_l^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} . \quad (11)$$

In order to compare our calculations with experimental results, we need the neutrino mass matrix at  $M_Z$  scale, which is obtained from the following one-loop RGE's relation between the neutrino mass matrices at  $m_Z$  and  $M_R$  [21];

$$\tilde{M}_\nu(M_Z) = \begin{pmatrix} \frac{1}{1-\epsilon_e} & 0 & 0 \\ 0 & \frac{1}{1-\epsilon_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tilde{M}_\nu(M_R) \begin{pmatrix} \frac{1}{1-\epsilon_e} & 0 & 0 \\ 0 & \frac{1}{1-\epsilon_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad (12)$$

where  $\tilde{M}_\nu$  is the neutrino mass matrix on the basis where charged lepton matrix is diagonalized (see Eq. (10)). The renormalization factors  $\epsilon_e$  and  $\epsilon_\mu$  depend on the ratio of VEV's,  $\tan\beta_v$ . By using the form of Eq. (12) we search the region of the parameter set  $(\alpha, \beta, h, \phi, \sigma, \rho)$  which are allowed by experimental data within  $3\sigma$ :

$$\begin{aligned} 0.82 &\leq \sin^2 2\theta_{\text{atm}} , \\ 0.28 &\leq \tan^2 \theta_{\text{sun}} \leq 0.64 , \\ 0.73 \times 10^{-3} &\leq \Delta m_{\text{atm}}^2 \leq 3.8 \times 10^{-3} \text{eV}^2 , \\ 5.4 \times 10^{-5} &\leq \Delta m_{\text{sun}}^2 \leq 9.5 \times 10^{-5} \text{eV}^2 , \end{aligned} \quad (13)$$

which are derived from Eq. (1).

Figure 1 shows scatter plots of the allowed region of  $h, \phi$ , in which the neutrino experimental results of Eq. (13) are reproduced by choosing the value  $\alpha, \beta, \rho, \sigma$ . This shows clearly that  $h$  cannot be taken too large or too small;  $0.4 \leq h \leq 3.0$ .

Also it is interesting that the phase factor  $\phi$  should not become large, ( $|\phi| \leq 70^\circ$ ). This may be important since we have never had the information of the phases appearing in  $M_\nu$ , which is connected to the leptogenesis. Let us explore an example of the allowed

region of the parameters in  $(\alpha, \beta)$  plane for the typical value  $h = 1.3$ . The allowed region which is consistent with the experimental data Eq. (13) is shown in Fig.2, where  $\beta$  is allowed to be in both negative and positive.

So far we have investigated the region of the parameters appearing in the neutrino mass matrix of Eq. (3) and shown that the parameter region is restricted within narrow range by the present experimental data. Here we make a comment whether or not a certain GUT model is consistent with the bi-large mixing with present neutrino mass differences.

As an example, let us take a concrete model [9] with the simplest form of right-handed neutrino mass matrix with the phase-factored out diagonal matrix,  $P_R$ ,

$$M_R = P_R^T \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix} P_R \equiv m_R P_R^T \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_R. \quad (14)$$

This, with the form of 4-zero texture form of  $M_{\nu_D}$ , yields also texture-zero form Eq. (3) with the phase factored out by  $(M_{\nu_D})_{RL} = P_{\nu_D}^T (\overline{M}_{\nu_D})_{RL} P_{\nu_D}$ ,

$$\overline{M}_{\nu_D} = \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix} m_{\nu_D} \rightarrow M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & 2\frac{ab}{r} + c^2 & c(\frac{a}{r} + 1) \\ 0 & c(\frac{a}{r} + 1) & 1 \end{pmatrix} \frac{m_{\nu_D}^2}{m_R}, \quad (15)$$

where  $a$  and  $c$  are real numbers and  $b$  is complex one. We recognize that, in order to get large mixing angle  $\theta_{23}$ , the 23 element must be of the same order as the 33 element, namely  $c(\frac{a}{r} + 1) \sim 1$ . Since  $c \ll 1$ ,  $ca/r$  must be of order 1. Thus approximate form of  $M_\nu$  is

$$M_\nu \sim \begin{pmatrix} 0 & \beta & 0 \\ \beta & e^{i\phi}\alpha & h \\ 0 & h & 1 \end{pmatrix} \frac{m_{\nu_D}^2}{m_R}, \quad \begin{aligned} \beta &\sim \frac{a^2}{r}, \\ \alpha &\sim \frac{2ab}{r}, \\ h &\sim \frac{ca}{r}, \end{aligned} \quad (16)$$

which clearly shows that none of  $a, b, c$  is zero, namely 6-zero texture are already excluded by the experimental neutrino data <sup>§</sup>. Now, one example of the symmetric 4-zero texture with the Pati-Salam symmetry [9] provides us with the Dirac neutrino mass matrix at the

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<sup>§</sup>Here, we note that the 6-zero textures for the quark sector have been already ruled out by Ramond, Roberts and Ross [22].

$M_{\text{GUT}}$  scale under a simple assumption of the following Higgs configurations:

$$M_U = \begin{pmatrix} 0 & \mathbf{126} & 0 \\ \mathbf{126} & \mathbf{10} & \mathbf{10} \\ 0 & \mathbf{10} & \mathbf{126} \end{pmatrix} \rightarrow \overline{M}_{\nu_D} \simeq \begin{pmatrix} 0 & -3\frac{\sqrt{m_u m_c}}{m_t} & 0 \\ -3\frac{\sqrt{m_u m_c}}{m_t} & e^{i\phi}\frac{m_c}{m_t} & \sqrt{\frac{m_u}{m_t}} \\ 0 & \sqrt{\frac{m_u}{m_t}} & -3 \end{pmatrix} m_t, \quad (17)$$

accompanying the phase factor  $P_D$  in a same way as Eq. (16). By comparing Eq. (16) and Eq. (17) the parameters  $\alpha, \beta$  are expressed in terms of up-quark masses at the GUT scale. Thus, we can predict  $\alpha, \beta$  from the up-quark masses at the GUT scale,  $m_u = 0.36 \sim 1.28\text{MeV}$ ,  $m_c = 209 \sim 300\text{MeV}$ ,  $m_t = 88 \sim 118\text{GeV}$ , which are obtained taking account of RGE's effect to the quark masses at the EW scale [23].

We show the region of  $\alpha, \beta$  predicted from the model of Eq. (17) in figure 3, where  $h = 1.3$  and  $m_u = 0.36 \sim 1.28 \text{ MeV}$  are taken. The allowed region predicted from a neutrino mass matrix with two zeros of Eq. (3) in figure 2 and the region given by the up-quark masses are separated slightly as seen in figure 3 if we take the up quark mass at the GUT scale,  $m_u = 0.36 \sim 1.28\text{MeV}$ , seriously.

However the light quark masses are ambiguous because of the non-perturbative QCD effect. Therefore the allowed mass region of  $m_u$  may be enlarged. In the case of  $m_u = 0.36 \sim 2.56 \text{ MeV}$ , we obtain the overlapped region around  $\alpha \simeq 1.24$  and  $\beta \simeq -0.2$  with  $h = 1.3$  as seen figure 4. The allowed region on the  $\alpha - \beta$  plane in the case of  $h = 1.3$ , which is predicted from a neutrino mass matrix with two zeros of Eq. (3). The allowed region of the parameters are very narrow as follows:

$$\alpha = 1.23 \sim 1.24, \quad \beta = -0.199 \sim -0.197, \quad \phi = -\frac{\pi}{18} \sim \frac{\pi}{18}, \quad \rho = \frac{7}{9}\pi \sim \frac{11}{9}\pi, \quad (18)$$

where  $h = 1.3$  is taken. On the other hand, our results are almost independent of the phase parameter  $\sigma$ . Hereafter we take  $\sigma = 0$  in our calculations. In these parameters, we can predict  $U_{e3}$  by including the contribution of the charged lepton sector. Here we stress that  $U_{e3}$  is crucial to discriminate various models, therefore, we must be careful to estimate it by taking account of the effect of charged lepton mixings as well as CP violating phases. Our formula has already included these contributions. By taking the overlapped region of  $\alpha$  and  $\beta$  in figure 4, we present the prediction of  $|U_{e3}|$ ,  $J_{CP}$  and  $\langle m_{ee} \rangle$  as follows:

$$|U_{e3}| = 0.010 - 0.048, \quad |J_{CP}| \leq 9.6 \times 10^{-3}, \quad |\langle m_{ee} \rangle| \simeq 0.0027\text{eV}, \quad (19)$$

where  $\langle m_{ee} \rangle$  is the effective neutrino mass in the neutrinoless double beta decay. We hope  $|U_{e3}|$  can be checked by the neutrino experiments in near future. Since the overlapped region of  $\alpha$  and  $\beta$  is restricted in the narrow region, we can predict a set of typical values of neutrino masses and mixings at  $h = 1.3$  as follows;

$$\begin{aligned} \sin^2 2\theta_{\mu\tau} &\sim 0.98, & \tan^2 \theta_{\mu e} &\sim 0.28, \\ m_{\nu_3} &\sim 0.062 \text{ eV}, & m_{\nu_2} &\sim 0.0075 \text{ eV}, & m_{\nu_1} &\sim 0.0014 \text{ eV}, \end{aligned} \quad (20)$$

with  $m_R = 3.0 \times 10^{15} \text{ GeV}$  and  $r \times m_R = 1.0 \times 10^9 \text{ GeV}$ , which correspond to the Majorana mass for the third generation and those of the second and first generations, respectively. On the other hand,  $m_u \simeq 2.56 \text{ MeV}$  should be allowed at the GUT scale. Now that our neutrino mass matrix is determined almost uniquely from the up-quark masses at GUT scale, we can make the prediction of leptogenesis once we fix the CP violating phases. Interesting enough is that our form of  $M_R$  of Eq. (14) yields naturally two degenerate Majorana masses with mass  $r \times m_R \sim 10^9 \text{ GeV}$ . In such case the leptogenesis is enhanced by the so-called "crossing effect" [24], which are now under calculation by Bando, Kaneko, Obara and Tanimoto[25].

In conclusion we have shown that, in order to be compatible with the present neutrino experiments, the parameters of a neutrino mass matrix with two zeros in Eq. (3) are constrained to a small region. Also, we have seen that the 4-zero texture with Pati-Salam symmetry restricts the above parameter region to a very narrow region indicated in Figure 4, enlarging the values of up quark mass at the GUT scale. Both parameter regions should be compared in detail, which will be published elsewhere in the near future. The precision measurements, especially, for the solar neutrino mixing angle and the mass squared differences will check if such a texture of geometric form with Pati-Salam symmetry is realized in Nature in the near future.

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## References

- [1] Super-Kamiokande Collaboration, Y. Fukuda et al., **Phys. Rev. Lett.** **81**, 1562 (1998); *ibid.* **82**, 2644 (1999); *ibid.* **82**, 5194 (1999);  
K. Nishikawa, Invited talk at XXI Lepton Photon Symposium, August 10-16, 2003, Batavia, USA.
- [2] Super-Kamiokande Collaboration, S. Fukuda et al., **Phys. Rev. Lett.** **86**, 5651; 5656 (2001).
- [3] SNO Collaboration: Q. R. Ahmad et al., **Phys. Rev. Lett.** **87**, 071301 (2001), *ibid.* **89**, 011301 (2002), *ibid.* **89**, 011302 (2002), nucl-ex/309004.
- [4] KamLAND Collaboration, K. Eguchi et al., **Phys. Rev. Lett.** **90**, 021802 (2003).
- [5] G. L. Fogli, E. Lisi, M. Marrone, D. Montanino, A. Palazzo and A.M. Rotunno, **Phys. Rev. D****67**, 073002 (2003);  
J. N. Bahcall, M. C. Gonzalez-Garcia and C. Peña-Garay, **JHEP** **0302**, 009 (2003);  
M. Maltoni, T. Schwetz and J.W.F. Valle, **Phys. Rev. D****67**, 093003 (2003);  
P.C. Holanda and A. Yu. Smirnov, **JCAP** **0302**, 001 (2003);  
V. Barger and D. Marfatia, **Phys. Lett.** **555B**, 144 (2003);  
M. Maltoni, T. Schwetz, M. Tórtola and J.W.F. Valle, hep-ph/0309130.
- [6] CHOOZ Collaboration, M. Apollonio et al., **Phys. Lett.** **B466**, 415 (1999).
- [7] L. Ibáñez and G. G. Ross, **Phys. Lett.** **332B**, 100 (1994);  
P. Binétruy, S. Lavignac and P. Ramond, **Phys. Lett.** **350B**, 49 (1995); **Nucl. Phys.** **B477**, 353 (1996);  
J. K. Elwood, N. Irges and P. Ramond, **Phys. Rev. Lett.** **81**, 5064 (1998);  
N. Irges, S. Lavignac and P. Ramond, **Phys. Rev. D****58** (1998) 035003;  
J. Sato and T. Yanagida, **Phys. Lett.** **B430**, 127 (1998);  
M. Bando and T. Kugo, **Prog. Theor. Phys.** **101**, 1313 (1999).
- [8] M. Bando and M. Obara, hep-ph/0305016.
- [9] M. Bando and M. Obara, **Prog. Theor. Phys.** **109**, 995 (2003).
- [10] M. Bando and T. Kugo, hep-ph/0308258.

- [11] H. Nishiura, K. Matsuda and T. Fukuyama, **Phys. Rev. D****60**, 013006 (1999);  
K. Matsuda and T. Fukuyama and H. Nishiura, **Phys. Rev. D****61**, 053001 (2000).
- [12] P.H. Frampton, S.L. Glashow and D. Marfatia, **Phys. Lett. B****536**, 79 (2002).
- [13] Z. Xing, **Phys. Lett. B****530**, 159 (2002).
- [14] A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, **Phys. Lett. B****538**, 96 (2002).
- [15] R. Barbieri, T. Hambye and A. Romanino, hep-ph/0302118.
- [16] M. C. Chen and K. T. Mahanthappa, **Phys. Rev. D****68**, 017301(2003).
- [17] Z. Maki, M. Nakagawa and S. Sakata, **Prog. Theor. Phys.** **28**, 870 (1962).
- [18] A. Yu. Smirnov, **Phys. Rev. D****48**, 3264 (1993);  
M. Tanimoto, **Phys. Lett. B****345**, 477 (1995);  
M. Bando, T. Kugo and K. Yoshioka, **Phys. Rev. Lett.** **80**, 3004 (1998).
- [19] See the Nishiura, Matsuda and Fukuyama in reference [11] for the matrix form of  $O_l$ .
- [20] H. Georgi and C. Jarlskog, **Phys. Lett. B****86**, 297 (1979) .
- [21] N. Haba, Y. Matsui, N. Okamura and M. Sugiura, **Eur. Phys. J. C****10** (1999) 677;  
**Prog. Theor. Phys.** **103** (2000) 145;  
N. Haba and N. Okamura, **Eur. Phys. J. C****14** (2000) 347;  
N. Haba, N. Okamura and M. Sugiura, **Prog. Theor. Phys.** **103** (2000) 367.
- [22] P. Ramond, R.G Roberts and G.G Ross, **Nucl. Phys. B****406**, 19 (1993).
- [23] H. Fritzsch and Z-z. Xing, **Prog Part Nucl. Phys.** **45**, 1 (2000).
- [24] E. A. Akhmedov, M. Frigerio and A. Yu. Smirnov, hep-ph/0305322.
- [25] M. Bando, S. Kaneko, M. Obara and M. Tanimoto, in preparation.

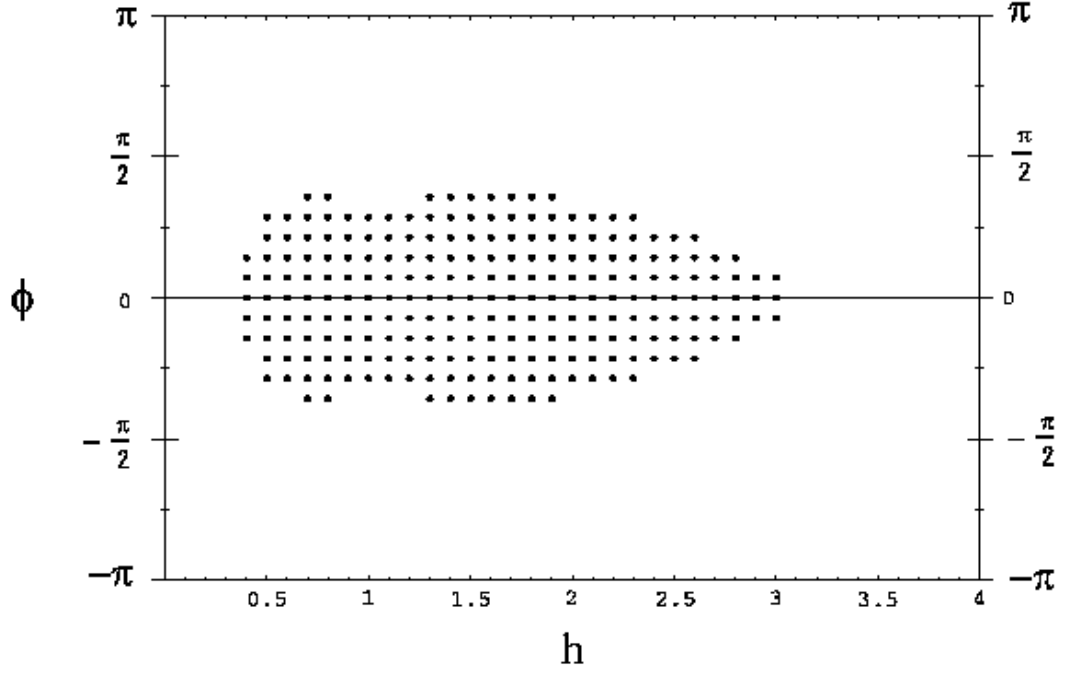


Figure 1: The scatter plots of the allowed region on the  $h - \phi$  plane.

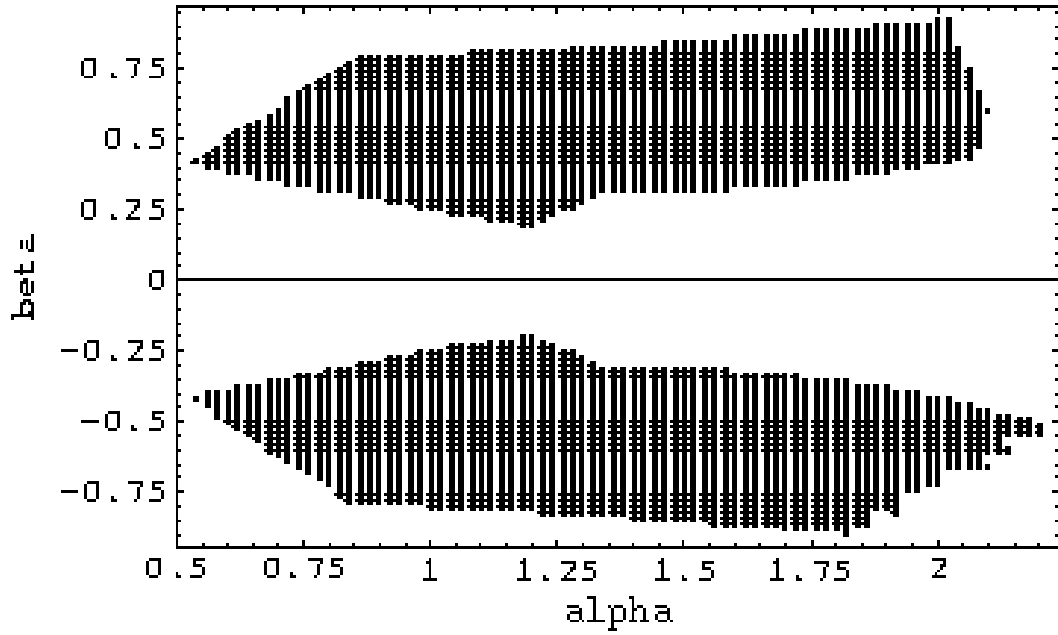


Figure 2: The allowed region on the  $\alpha - \beta$  plane in the case of  $h = 1.3$ , which is predicted from a neutrino mass matrix with two zeros of Eq. (3).

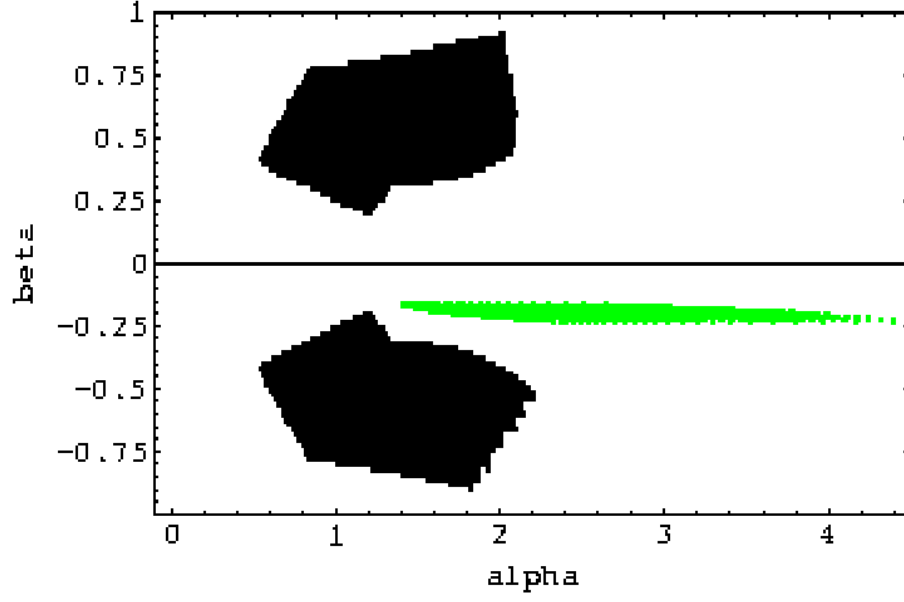


Figure 3: The predicted region (gray region) of the  $\alpha - \beta$  plane in the GUT model, where  $h = 1.3$  and  $m_u = 0.36 \sim 1.28$  MeV are taken. The black region is the experimentally allowed region predicted from a neutrino mass matrix with two zeros of Eq. (3).

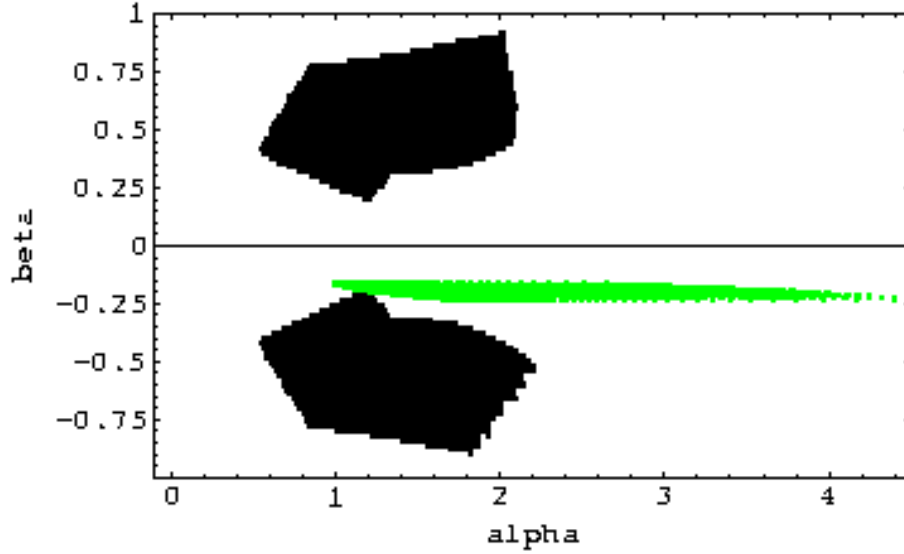


Figure 4: The predicted region (gray region) of the  $\alpha - \beta$  plane, in which  $h = 1.3$  and  $m_u = 0.36 \sim 2.56$  MeV are taken. The black region is the experimentally allowed region predicted from a neutrino mass matrix with two zeros of Eq. (3). There is the overlapped region around  $\alpha \simeq 1.24$  and  $\beta \simeq -0.2$ .